Keywords Venable, frequency response analyzers, power supply circuits, control to output, transfer function, error amplifier, filter inductor, filter capacitor, parasitic inductance, open loop gain, output impedance

APPLICATION NOTE:

Using CAE/CAT Tools To Evaluate Power Supply Parasitics

Abstract:

Parasitic elements do not show up on the schematic or parts list! Even though much attention has been focused recently on modeling of power supply circuits, that one major drawback has greatly diminished the success and effectiveness of modeling. Values of parasitic elements are not easily determined. This paper describes the application of computer-aided engineering and computer-aided test equipment, which can identify and quantify these parasitic elements. Actual test data are used to determine component values in a model where five of the eight components are parasitic elements.

Introduction

During a recent consulting assignment, we needed to model a magnetic amplifier (magamp) post-regulator. Magamp post-regulators are particularly difficult circuits to model. There are eight elements in the canonical model of even the simplest of these circuits, and five of the eight are parasitics. In developing the techniques for determining the values of these parasitics, we learned lessons, which we found valuable. The purpose of this paper is to share those lessons and techniques with others who may have similar needs or interests.

Figure 1 shows a schematic diagram of a magamp post-regulator. This particular circuit is for a negative output voltage. This circuit can be further divided into a power-processing portion which we call the "modulator," and an error amplifier portion, which we call the "amplifier."

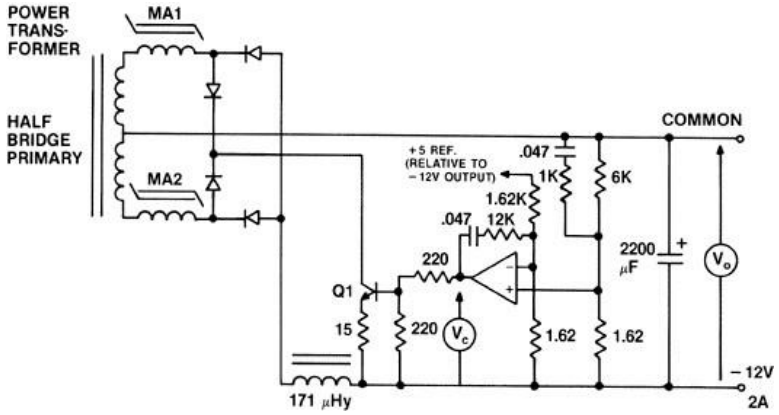

Figure 1. Schematic Diagram of Magamp Post-regulator

The amplifier is relatively easy to model accurately. This process will be shown, but not in elaborate detail. The really difficult portion of the circuit to model is the modulator, for this is the portion that has almost all the significant parasitic elements. This part we built and tested, and used the test data to determine the actual values of the parasitics.

Consider the circuit of Figure 1. The modulator is the portion from the control voltage, Vc, to the output voltage, Vo. A model of this portion of the circuit useful for spice-type modeling is shown in Figure 2. The components of the model of the control-to-output transfer function are:

1. The DC gain (a parasitic related to core characteristics)
2. A time delay (a parasitic, roughly one switching period)
3. Series resistive loss (a parasitic, sum of series element losses)
4. Leakage and stray inductance (sum of several parasitic inductances)
5. The filter inductor (a real component, at last!)
6. The filter capacitor (another real component)
7. The ESR of the filter capacitor (a parasitic)
8. The load resistor (usually a known component)

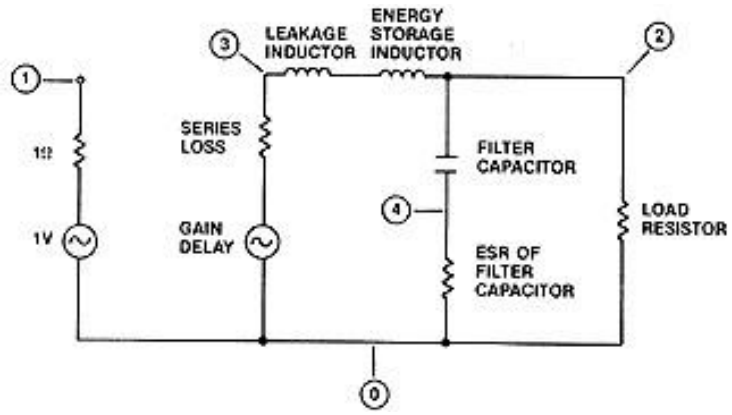

Figure 2. Control-to-Output Spice Model

Parasitic Versus Real Components

DC Gain

Let us examine each of these elements in more detail. There is DC gain from the control voltage Vc (node 1) to the output voltage Vo (node 2). This is a parasitic. The control voltage sets a particular current through the magamp during the reset portion of the cycle because of current source transistor Q1. The current is determined, but the critical parameter in the magamp is flux, not current. Flux is proportional to volt-seconds, and voltage is created by loss currents flowing in the resistivity of the core material. All these parameters lend themselves more to measurement than to calculation and prediction. It is flux stored in the magamp during the reset cycle that becomes blocking time during the power cycle, and ultimately determines the duty cycle of the waveform applied to the energy storage inductor.

Time Delay

There is also a time delay, since data are stored in the magamp in the form of flux on one cycle of the waveform, and delivered as pulse width on the next cycle. The time delay is roughly but not exactly the period of one cycle. This parameter is also more easily measured than calculated or predicted.

Series Resistive Losses

The series resistive loss is the resistance of all the transistors, windings, diodes, chokes, and wiring, reflected to the secondary side of the transformer. Some of these resistances are dynamic, that is, they are a function of the operating point (primarily load current). While it may be possible to make a calculated guess about most of them, the cumulative value is very difficult to calculate accurately.

Leakage and Stray Inductance

The leakage and stray inductance are difficult to calculate also. A typical power supply consists of a half-bridge primary and magamp post-regulators on the auxiliary outputs. The leakage and stray inductance is primarily the power transformer primary leakage inductance reflected to the secondary (the secondary leakage inductance), and the saturated inductance of the magamp. These are difficult to measure and almost impossible to calculate since they depend heavily on the lay of the magnet wire on the core. The stray inductance of the wiring is usually ignored since it is small and difficult to calculate.

Energy Storage Inductor

At last, we come to a real component: the filter inductor. This part is usually fabricated rather than purchased. The value of this component can be measured, and is sometimes actually given on the schematic. There is normally a slight variation of value with operating current, but even this can be measured if necessary and the correct value entered in the model.

Filter Capacitor

The filter capacitor is almost always a purchased component. The value is printed on the part, and sometimes the tolerance also. If not, the tolerance can be obtained from the data sheet. The actual value can also be measured. This is the easiest of the eight parts to obtain a value for.

ESR of Filter Capacitor

The equivalent series resistance (ESR) of the filter capacitor is another parasitic element. It is never on the schematic, but an approximate value is sometimes printed in the capacitor data sheet. ESR used to be given only as a maximum value, usually 10 times the actual value, but the ESR value is so critical to the operation of a power supply circuit that some manufacturers now give a value with a relatively tight tolerance such as ±20%.

Load Resistance

The load resistance is normally known, at least to the power supply manufacturer who has built a resistive load bank to test the supply. The customer may not know the true load resistance until the design of the using equipment is complete. This is often after the manufacturer has shipped the power supply to the customer.

There are other parasitic or difficult to measure components, such as load capacitance or the inductance of power leads when using remote sensing. We are not going to address these components, but their values could be calculated by following steps similar to the ones we are presenting.

Calculating the Parasitic Element Values

The first step in calculating the values of the parasitic elements is to measure the transfer function from the control voltage Vc to the output voltage Vo. These data, plus the values of the three known components, contain all information necessary to evaluate the five parasitic components. We used a Venable Industries Model 350 Frequency Response Analyzer System for the measurements and the subsequent calculations.

The transfer function from control to output is shown in Figure 3. This is actual test data obtained recently [at the time of writing] on a commercial power supply for a computer application. The DC gain is slightly over 23 dB. There is an L-C corner at approximately 200 Hz, with some peaking of the gain curve. An ESR zero from the resistance of the filter capacitor causes the gain slope to change from –2 (12 dB/octave) to –1 (6 dB/octave) at roughly 1.5 kHz. The ESR zero frequency was obtained by guessing where the gain slope transitions from –2 to –1. (The intersection of the asymptotic –2 and –1 portions of the gain curve.) Determining this frequency accurately is not critical at this point. A more accurate value will come from the analysis, and if the initial guess was too far off, an iteration of the calculations can be done to improve the result.

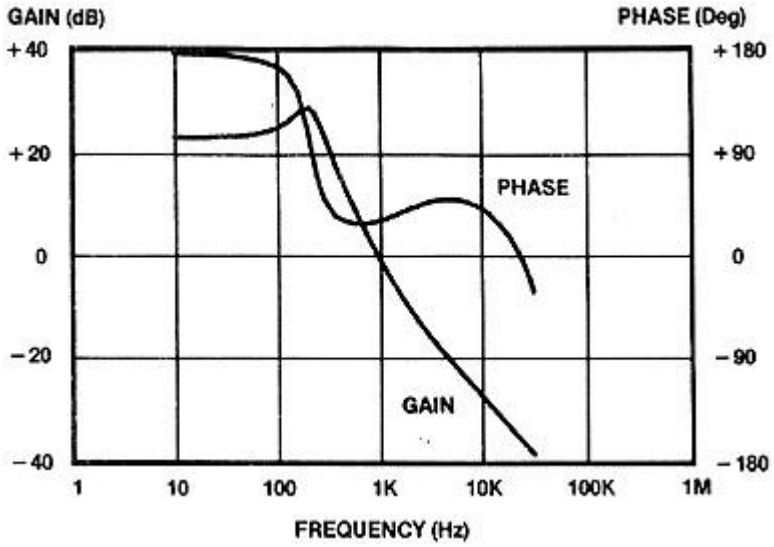

Figure 3. Control-to-Output Transfer Function

The phase curve starts at +180° because this particular circuit inverts at DC. The curve then transitions rapidly toward zero at the L-C corner frequency. Phase shift then starts to transition upward toward +90° because of the ESR zero, only to head rapidly negative soon thereafter as the phase lag due to magamp time delay becomes significant.

Determining the Time Delay

The first step in the process of parasitic component evaluation is to determine the time delay imposed by the magamp. This is done at the highest frequency for which accurate data are available, in this case 30 kHz. The phase shift at 30 kHz, which was read from the graphics display, is –29.1°. The residual phase shift from the ESR zero can be calculated from the formula:

> Residual phase shift = tan–1(fz/ f) where fz is the ESR zero frequency and f is the highest measured frequency. In this example, fz = 1.5 kHz and f = 30 kHz, so:
>
> Residual phase shift = tan–1(1.5 / 30) = 2.86 degrees.

This is the difference between actual phase shift and 90° which results from the ESR zero if a time delay were not present. Since the poles are at a much lower frequency than the zero, phase shift is assumed to be negligible. The phase shift due to time delay can then be calculated from:

> Delay phase shift = (90 – residual) – (measured)

where "residual" means residual phase shift from the ESR zero as calculated above, and "measured" means measured phase shift at the highest data point frequency. In this example, residual = 2.86 degrees and measured = –29.1 degrees, so:

> Delay phase shift = (90 – 2.86) – (–29.1) = 116.24 degrees.

This phase shift needs to be converted to time. One cycle is always 360 degrees, and at 30 kHz this takes 1/30,000th of a second, which is 33.3 microseconds. The time delay is then given bythe formula:

Delay = Time delay phase shift * period 360

Here, time delay phase shift = 116.24 degrees and the period is 33.3 microseconds, so:

Delay = 116.24 * 33.3 360 = 10.8 microseconds.

The operating frequency of this particular power supply is 43.5 kHz, so the half period is 11.5 microseconds. You can see that the delay is close to but not exactly one-half period.

Remove the Time Delay from the Data

A necessary feature of the software is the ability to add or subtract time delays from data. Time delay is a phase shift, which increases linearly with frequency. Here, the amount of phase shift equivalent to a 10.8 microsecond delay must be subtracted from each frequency point in the measured phase shift potion of the data file. Figure 4 shows the control-to-output transfer function with the time delay removed.

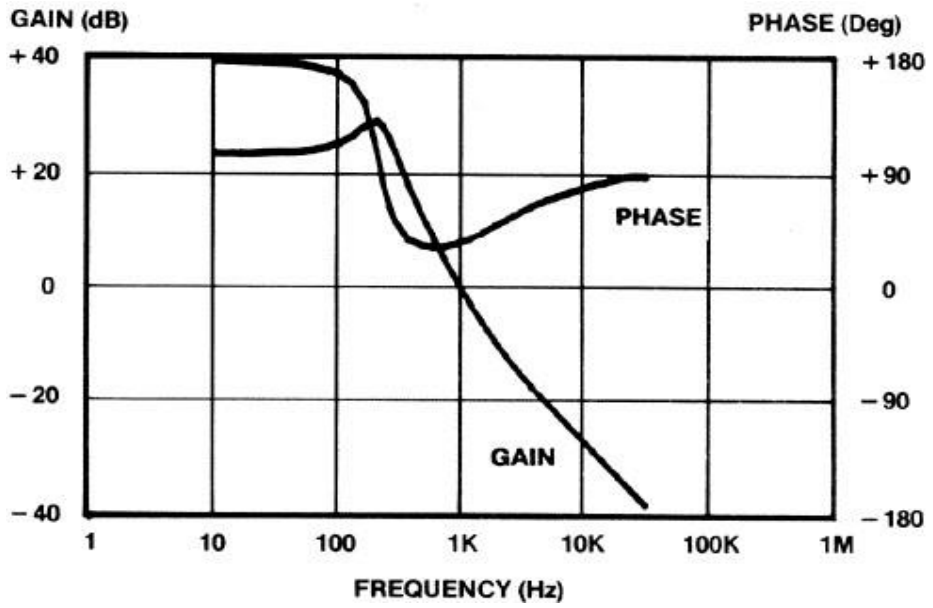

Figure 4. Test Data with Time Delay Removed

Time delays add more phase shift than can come from passive elements, and networks that have time delays are therefore called "non-minimum phase" networks. Although time delays can be simulated mathematically by a coincident right-half-plane zero and left-half-plane pole pair, these extra zeroes and poles tend to be confusing when trying to separate them from the real zeroes and poles which come from the passive components in the network. A much better way to deal with time delays is to remove them, as in this example, to achieve a network without time delays. This type of network is

called a "minimum phase" network. By removing the time delay, we have transformed a non-minimum-phase network into a minimum-phase network, and the zeroes and poles of the network can be much more easily evaluated.

Locating the Zeroes and Poles

Another necessary feature is the ability to find the zeroes and poles of a transfer function. Some frequency response analyzers have this feature built in. Others, like the one we used, do the analysis in software on a data file. The zero-pole factoring software was used on the minimum phase data file obtained from the previous step, assuming 1 zero and 2 poles. The following results were obtained:

Gain = –14.56
Zero = –1.54 kHz
Poles = –52.94 ± 203.48 Hz

The gain of –14.56 means the circuit inverts at DC. A gain of 14.56 translates to 23.26 dB, so it agrees with the plot. The dimensionless gain of 14.56 will be used in the calculations for the model.

The negative sign of the zero and poles means they are conventional left-half-plane zeroes and poles, the result of passive components.

Finding the Capacitor ESR

The first use of the result of the mathematical analysis is to find the equivalent series resistance of the filter capacitor. This capacitor has a value of 2200 microfarads, and the ESR zero frequency comes directly from the analysis as 1.54 kHz. If we call this frequency f, the value of ESR comes directly from the formula:

$$ESR = \frac{1}{2 * pi * f * C}$$

where C is the value of the capacitor. In this case,

$$ESR = \frac{1}{2 * 3.1416 * 1540 * 0.0022} = 47 \text{ milliohms}$$

Calculating the Parasitic Inductance

The parasitic inductance is in series with the real inductance. Its value is determined by its effect on the resonant frequency of the L-C circuit. The natural resonant frequency is the vector sum of the real and imaginary parts of the poles found above in section 3.3. This frequency can be found in the formula:

Natural resonant frequency = Sqrt (Re2 + Im2)

or in words: The natural resonant frequency equals the square root of the sum of the squares of the real and imaginary parts of the pole locations. In this example, the real part is 52.94 and the imaginary part is 203.48, so:

Natural resonant frequency = Sqrt (52.942 + 203.482) = 210.25 Hz.

The inductance required to resonate with 2200 microfarads at 210.25 Hz is given by the formula:

Effective inductance = 1 / ((2 * pi * f) 2 * C)

where f is the natural resonant frequency and C is the value of the resonating capacitor. In this example, f is 210.25 Hz and C is 2200 microfarads, so:

Effective inductance = 1 / ((2 * 3.1416 * 210.25) 2 * .0022) = 260 uHy.

This is the sum of the parasitic inductance and the energy storage inductor, whose value is 171 microhenries. The parasitic inductor value is then the difference between these values:

Parasitic inductance = 260 uHy – 171 uHy = 89 uHy.

Calculating the Parasitic Series Resistance

The Q of the L-C resonant circuit is controlled by the resistances, that is the series loss resistance, the ESR, and the load. The load is given, 12 volts at 2 amps, yielding a resistance of 6 ohms. In order to calculate the series loss resistance, the Q and the characteristic impedance of the L-C circuit must be calculated. The Q is calculated from the pole locations. The first step is to find the damping factor, "zeta," and then use zeta to calculate Q.

Zeta =
Real Pole Frequency
Natural resonant frequency

In this example, the real portion of the pole frequency is 52.94 and the natural resonant frequency is 210.25, so:

Zeta = 52.94
210.25 = 0.25

The equation for Q is:

Q = 1
2 * zeta

Therefore:

Q = 1 2 * 0.25 =
2.

The characteristic impedance of the L-C circuit is the impedance of either the L or C when the impedances are equal, which happens at resonance. The characteristic impedance is called Zo. The formula for Zo is:

Zo = Sqrt (L C)

L is the effective inductance, 260 microhenries, and C is the value of the filter capacitor, 2200 microfarads, so:

Zo = Sqrt (260 2200) = 0.344 ohms.

The effective series damping resistance of the L-C circuit can be calculated from Zo and Q. When Q = 1, the series damping resistance is equal to the characteristic impedance, Zo.

The important thing is the relationship of the various resistances relative to the characteristic impedance.

In this example, the load resistor (6 ohms) is much smaller. The load resistor (6 ohms) is much larger than the characteristic impedance (0.344 ohms), and the ESR (0.047 ohms) is much smaller. The load resistor can be reflected as a series loss element for damping purposes, with a value inversely proportional to its value relative to the characteristic impedance. The formula for calculating the extra resistor in series with the power path is then:

Series loss resistor = Zo/ Q – ESR – (Zo) 2/ RL

where ESR is the equivalent series resistance of the filter capacitor and RL is the load resistor. This equation says that the extra series loss resistor is the total damping resistance (Zo/ Q), minus the internal resistance of the capacitor, and minus the load resistor reflected as a series damping resistance.

In this example, Series loss resistance = 0.344 / 2 – 0.047 – (0.344) 2/6 = 0.105 ohms.

Calculating the Gain

The gain used in the model must consider both the effective DC gain, which results from the analysis and the divider ratio created by the series loss resistor and the load resistor. The effective DC gain calculated (dimensionless gain, not dB) is 14.56, the series loss resistor is 0.105 ohms, and the load resistor is 6 ohms. The model gain is then larger by the resistive divider ratio, as given by the formula:

Gain = Effective gain * ((RL + SLR) / RL)

where SLR is the series loss resistance, 0.105 ohms. The gain to use in the model is then:

Gain = 14.56 * ((6.105) / 6) = 14.81

This completes the evaluation of all the elements in the model, the five parasitic elements and the three discreet ones.

4. Putting the Model Together

Now that all the element values of the model are determined, the model can be completed. Figure 5 shows a schematic like Figure 2 with the values filled in. This in turn leads to a net list, which can be used in any spice-like modeling program. The net list for the model of Figure 5 is shown below:

```
1 V 0 0 1
2 R 1 0 1
3 V 0 1 14.81
4 R 3 0 .105
5 L 3 2 26OU
6 C 2 4 22OOU
7 R 4 0 .047
8 R 2 0 6
```

The sequence is branch number, component type, one connection or control node, the other connection or control node, and component value or gain.

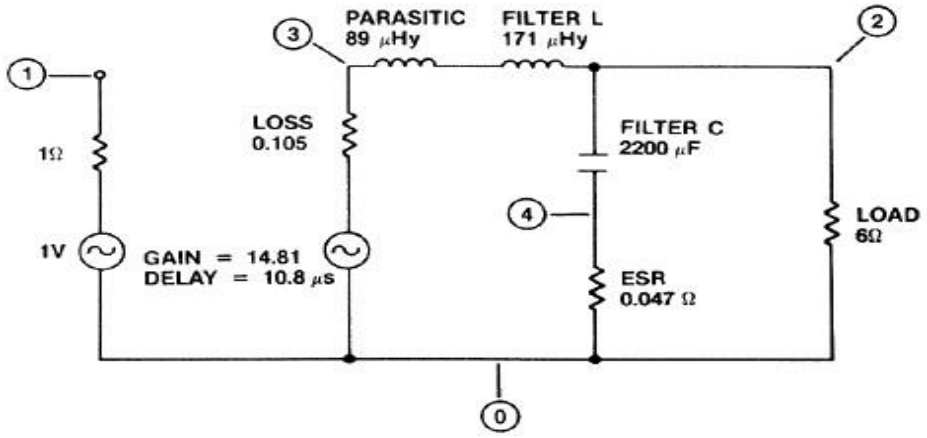

Figure 5. Model Schematic with Values

Each voltage source has a component type of V, and is followed by a source resistance ranch. The nodes following V are control nodes, and the nodes following the source resistance are the connection nodes. Control nodes of 0,0 represent a fixed source.

The first two branches represent a source such as a generator driving the circuit. Branches 3 and 4 are a controlled voltage source representing the gain of the circuit and the series loss element. This source is controlled by the voltage from 0 (ground) to 1, and its output is connected from node 3 to ground. The sequence of control and output nodes accomplishes the inversion present in the actual circuit. Branches 5 and 6 are the effective inductor and filter capacitor respectively, Branch 7 is the ESR of the filter capacitor, and Branch 8 is the load resistor.

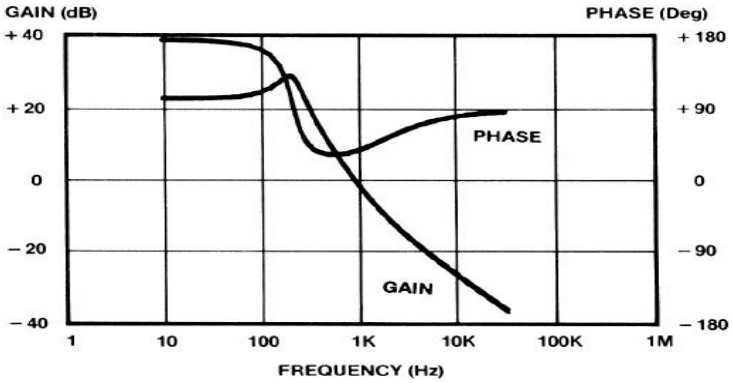

Figure 6. Model Transfer Function

This model can be analyzed to produce a minimum-phase transfer function as shown in Figure 6. After the model analysis, the time delay is added to the results. The 10.8 microsecond time delay is added to the minimum-phase analysis results, so the final transfer function matches the real world test results. The transfer function with time delay is shown in Figure 7. Figure 8 shows a Bode plot with the test data and final model data superimposed.

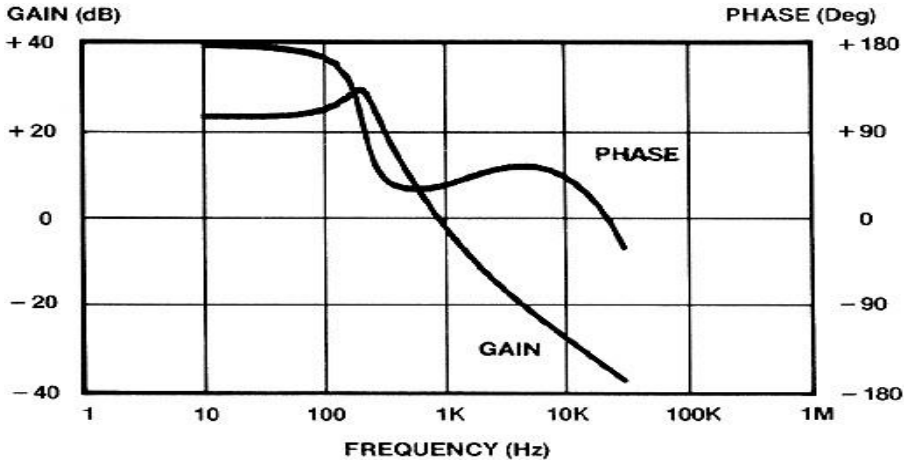

Figure 7. Model Transfer Function with Delay

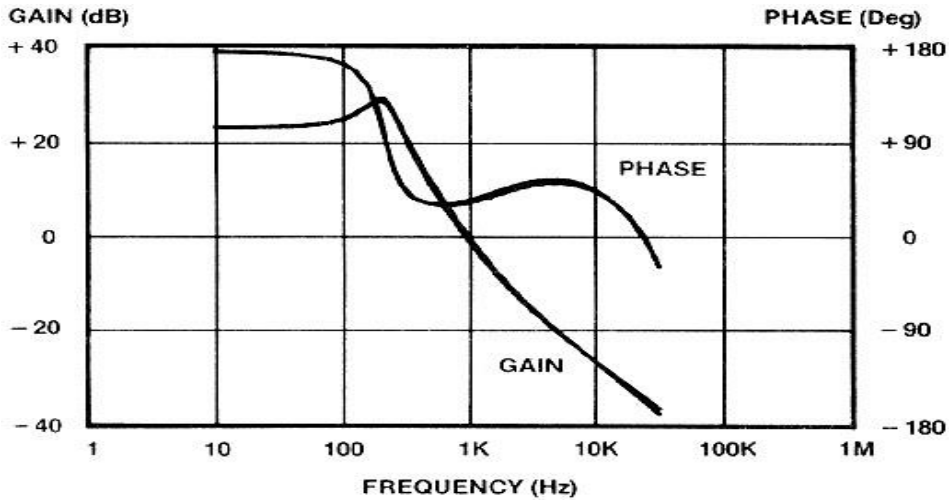

Figure 8. Test and Model Data Superimposed

Modeling the Amplifier

The amplifier is relatively straightforward to model. The only extra component not on the schematic is a capacitor whose purpose is to provide for the gain-bandwidth limitation of the opamp. The schematic of the amplifier is shown in Figure 1 and a model of the amplifier is shown in Figure 9.

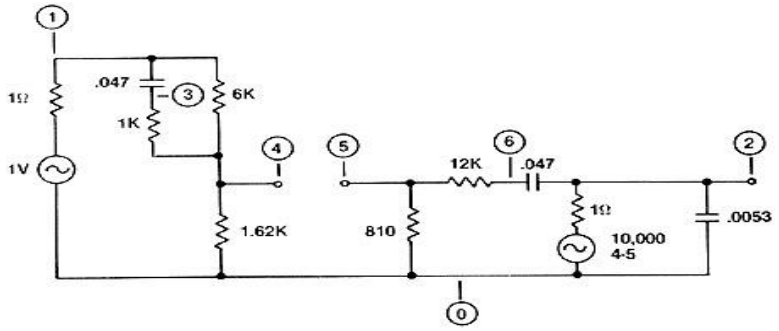

Figure 9. Model of Error Amplifier

The spice net list which results from the model is shown below:

1 V 0 0 1
2 R 1 0 1
3 C 1 3 .047U
4 R 3 4 1K
5 R 1 4 6K
6 R 4 0 1.62K
7 R 5 0 810
8 R 5 6 12K
9 C 6 2 .047U
10 V 4 5 10000
11 R 2 0 1
12 C 2 0 .0053

Branches 1 and 2 are again a fixed driving source. Branches 3-6 represent the voltage divider network connected to the non-inverting input of the op-amp. Branch 7 is the parallel combination of the two 1.62K resistors going to ground and the +5 reference. For AC analysis purposes, the reference voltage is DC and therefore the same as ground. Branches 8 and 9 are the feedback components from output

to the inverting input. Branches 10 and 11 represent the open loop gain of the op-amp, assumed to be 10000, and the capacitor in Branch 12 controls the open-loop gain to be unity at 300 kHz. An analysis of the amplifier circuit yields the Bode plot shown in Figure 10.

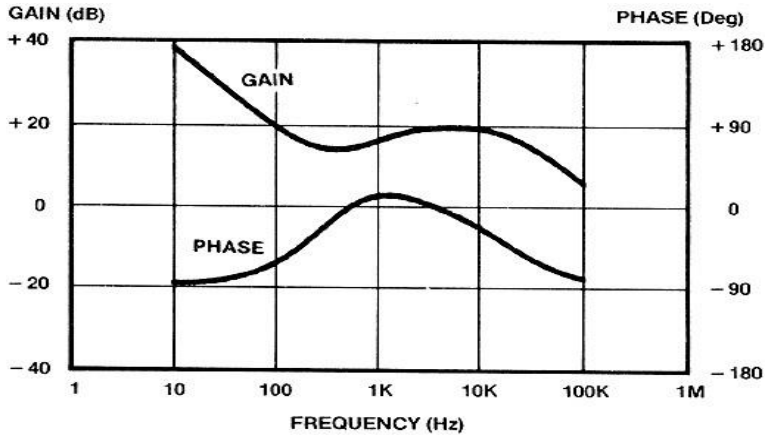

Figure 10. Transfer function of Amplifier

Closing the Loop

There are two ways to predict the loop gain. One is to combine the two models by changing node numbers and connecting the output of one to the input of the other and vice versa. Then an open loop plot can be made of the total circuit.

Another, easier way is to notice that the loop gain is the product of the amplifier gain and modulator gain, and to use software to multiply the two transfer functions together to predict the loop without the tedious process of changing node numbers. This is the process we used; the transfer functions were already calculated and available as data files. Figure 11 shows the results. The loop crosses over at 4.25 kHz w/ 51.6 degrees of phase margin. The gain margin is 11.55 dB.

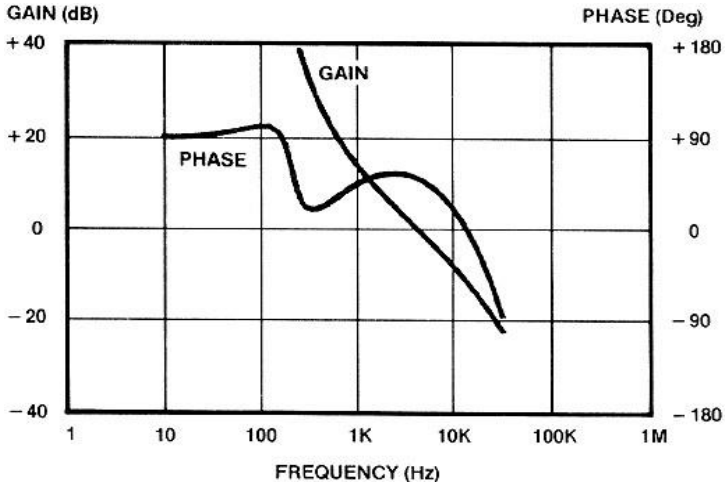

Figure 11. Predicted Open-Loop Gain

If another type of data is needed from the model — for example, transient data from step loading — it is necessary to combine the modulator and amplifier models as described above, and then drive the model as appropriate for the particular transfer function to be measured.

Summary

Techniques are developed for determining the values of the parasitic components, which influence the transfer function of most power processing circuits. The techniques are straightforward and numerical examples are given for clarity and as an aid for those who want to apply the methods.

A magamp post-regulator is used as an example, but the general shape of the power-processing transfer function is similar for most types of converters, and the techniques developed for determining the values of the parasitic components can be used for any type of converter topology, not only magamp post-regulators.

The model can also be used for evaluating other parameters of the design, such as transient load response or output impedance. When necessary, given the model, more intelligent choices can be made about design modifications required to achieve a desirable transfer function.

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